Modeling Data Distributions: Part 2

- Topics: Empirical Rule and Normal Distributions

- Objective: Students will be able to interpret data percentiles, calculate areas and percentages under normal distributions.

- Standards: AP Stats: VAR-2 (EU), VAR-2.A (LO), VAR-2.A.3 (EK)

Normal Distributions/Empirical Rule

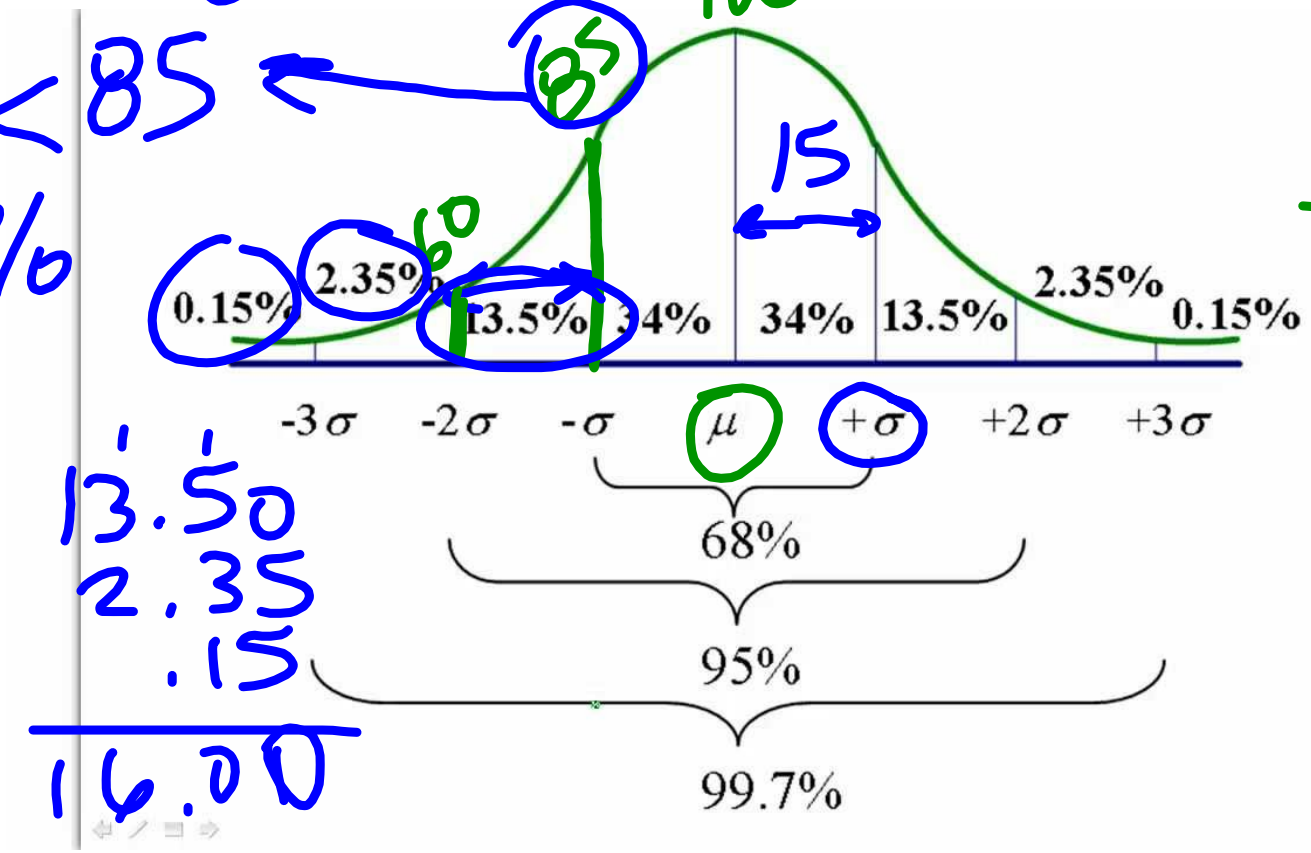
population

μ = median + mean

σ = Standard Deviation

$x < 85$

16%

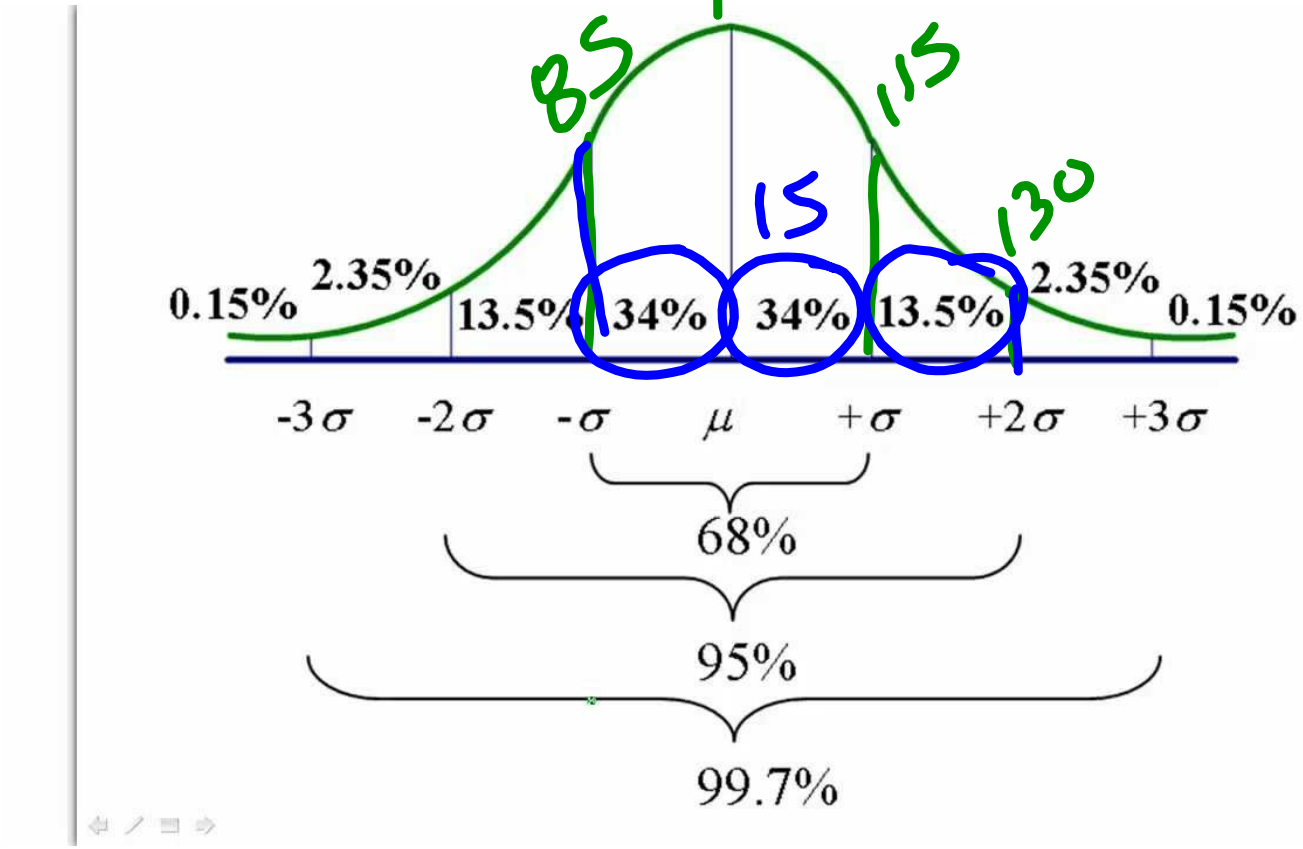


13.50
2.35
.15
16.00

100
- 15
85
- 15
60

Normal Distributions/Empirical Rule

$85 < \hat{x} < 130$



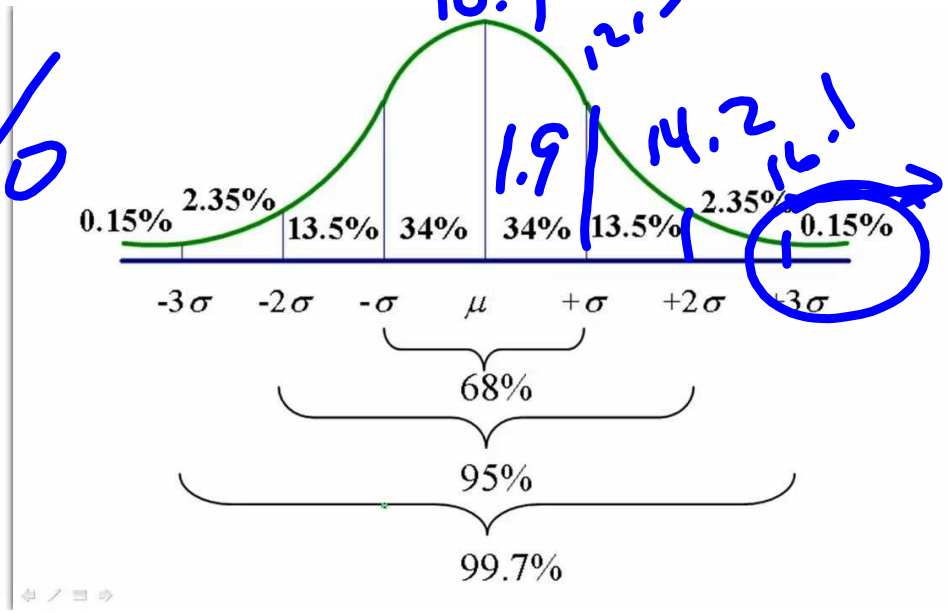
100
+15
115
+15
130

1
34
34
13.5
81.5

Normal Distributions/Empirical Rule

- Example: The lifespans of meerkats in a particular zoo are normally distributed. The average meerkat lives 10.4 years; the standard deviation is 1.9 years.
- *Use the empirical rule (68–95–99.7%) to estimate the probability of a meerkat living longer than 16.1 years.*

0.15%



10.4
+1.9
12.3
+1.9
14.2
+1.9
16.1

Normal Distributions: Finding Area

- Example: A set of piano prices are normally distributed with a mean of 3000 dollars and a standard deviation of 200 dollars. An electric piano has a price of 2576 dollars.
- *What proportion of piano prices are **higher** than the price of the electric piano?*
- *You may round your answer to four decimal places.*

1. *Calculate z-score*
2. *Look up in z-score table*
3. *If looking for **higher,** 1 minus (z-score result).*

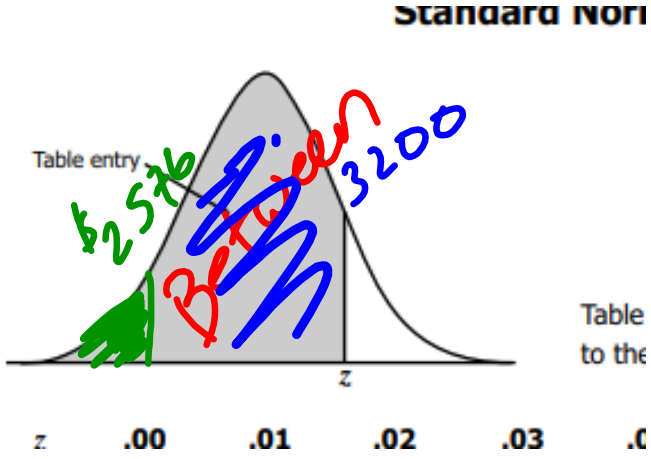
$z = \frac{\text{score} - \text{mean}}{\sigma} = \frac{2576 - 3000}{200}$

Lower

$z = -2.12$ $1 - .0170 = .9830$

Normal Distributions: Finding Area Between

- Example: A set of piano prices are normally distributed with a mean of 3000 dollars and a standard deviation of 200 dollars. Two electric pianos have a price of 2576 dollars and 3200 respectively.
- *What proportion of piano prices are **between** the two prices of the electric pianos?*
- *You may round your answer to four decimal places.*



$$\frac{3200-3000}{200}=1$$

.8413
−.0170
.8243

Z-Score Tables

Standard Normal Probabilities

Lower

-3.15

.0008

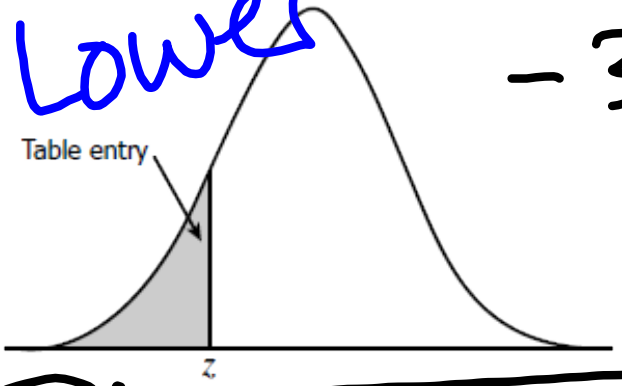


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143

Normal Calculations in Reverse

- Example: The distribution of SAT scores of all college-bound seniors taking the SAT in 2014 was approximately normal with mean μ=1497 and standard deviation σ=322.
- A certain test-retake preparation course is designed for students whose SAT scores are in the lower 25% of those who take the test in a given year.
- What is the maximum SAT score in 2014 that meets the course requirements?

1. Find z-score proportion of 25%. (Biggest without going over).
2. Put pieces into z-score formula and solve for the score.

Normal Calculations in Reverse

- μ=1497 and σ=322.
- ***lower*** 25% = .2500 look up on z-table
- What is the maximum SAT score in 2014 that meets the course requirements?

1. Find z-score proportion of 25%. (Biggest without going over).
2. Put pieces into z-score formula and solve for the score.

$.2500 \sim .2483 \Rightarrow$ z-score: -0.68

$$z = \frac{\text{score} - \mu}{\sigma} = -0.68 = \frac{x - 1497}{322}$$

$$1278 = x$$

Normal Calculations in Reverse

- μ=1497 and σ=322.
- ***Upper*** 25%, same as ***lower*** 75%
- What is the maximum SAT score in 2014 that meets the course requirements?

1. Find z-score proportion of 75%. (Biggest without going over). look for .7500 = z-score of 0.67
2. Put pieces into z-score formula and solve for the score.

$$z = \frac{score - \mu}{\sigma} \qquad 0.67 = \frac{x - 1497}{322}$$

$$1712.74 = x$$

Displaying and Comparing Quantitative Data

You should be working on the following skills:

1. Empirical rule
2. Normal distribution: Area above or below a point
3. Normal distribution: Area between two points
4. Normal calculations in reverse

Attachments

Ztable.pdf